# Higher Derivative Gravitation in Superstrings

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**Abstract**

A discussion of the number of degrees of freedom, and their dynamical properties, in higher derivative gravitational theories is presented. It is shown that non-vanishing $(C_{mnpq})^2$ terms arise in N=1, D=4 superstring Lagrangians due to one-loop radiative corrections with light field internal lines.

## 1 Bosonic Gravitation

The usual Einstein theory of gravitation involves a symmetric tensor $g_{\mu\nu}$ whose dynamics is determined by the Lagrangian

$$\mathcal{L} = -\frac{1}{2\kappa^2}\mathcal{R} \tag{1}$$

The diffeomorphic gauge group reduces the number of degrees of freedom from ten down to six. Einsteins equations futher reduce the degrees of freedom to two, which correspond to a physical spin-2 massless graviton. Now let us consider an extension of Einstein's theory by including terms in the action which are quadratic in the curvature tensors. This extended Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2\kappa^2}\mathcal{R} + \alpha\mathcal{R}^2 + \beta(C_{mnpq})^2 + \gamma(\mathcal{R}_{mn})^2 \tag{2}$$

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$\mathcal{R}^2$, $(C_{mnpq})^2$, and $(\mathcal{R}_{mn})^2$ are a complete set of CP-even quadratic curvature terms. The topological Gauss-Bonnet term is given by

$$GB = (C_{mnpq})^2 - 2(\mathcal{R}_{mn})^2 + \frac{2}{3}\mathcal{R}^2 \tag{3}$$

Therefore, we can write

$$\mathcal{L} = -\frac{1}{2\kappa^2}\mathcal{R} + a\mathcal{R}^2 + b(C_{mnpq})^2 + cGB \tag{4}$$

In this case, it can be shown [1] that there is still a physical spin-2 massless graviton in the spectrum. However, the addition of the $\mathcal{R}^2$ term introduces a new physical spin-0 scalar, $\phi$, with mass $M = \frac{1}{\sqrt{a}\kappa}$. Similarly, the $(C_{mnpq})^2$ term introduces a spin-2 symmetric tensor, $\phi_{mn}$, with mass $M = \frac{1}{\sqrt{b}\kappa}$ but this field, having wrong sign kinetic energy, is ghost-like. The GB term, being a total divergence, is purely topological and does not lead to any new degrees of freedom. The scalar $\phi$ is perfectly physical and can lead to very interesting new physics [2]. The new tensor $\phi_{mn}$, however, appears to be problematical. There have been a number of attempts to show that the ghost-like behavior of $\phi_{mn}$ is allusory, being an artifact of linearization [3]. Other authors have pointed out that since the mass of $\phi_{mn}$ is near the Planck scale, other Planck scale physics may come in to correct the situation [4]. In all these attempts, the gravitational theories being discussed were not necessarily consistent and well defined. However, in recent years, superstring theories have emerged as finite, unitary theories of gravitation. Superstrings, therefore, are an ideal laboratory for exploring the issue of the ghost-like behavior of $\phi_{mn}$, as well as for asking whether the scalar $\phi$ occurs in the superstring Lagrangian. Hence, we want to explore the question "Do Quadratic Gravitation Terms Appear in the N=1, D=4 Superstring Lagrangian"?

## 2 Superspace Formalism

In the Kahler (Einstein frame) superspace formalism, the most general Lagrangian for Einstein gravity, matter and gauge fields is

$$\mathcal{L}_E = -\frac{3}{2\kappa^2}\int d^4\theta E[K] + \frac{1}{8}\int d^4\theta \frac{E}{R} f(\Phi_i)_{ab} W^{\alpha a} W^b_\alpha + hc \tag{5}$$

where we have ignored the superpotential term which is irrelevant for this discussion. The fundamental supergravity superfields are R and $W_{\alpha\beta\gamma}$, which are chiral, and $G_{\alpha\dot{\alpha}}$, which is Hermitian. The bosonic $\mathcal{R}^2$, $(C_{mnpq})^2$ and $(\mathcal{R}_{mn})^2$ terms are contained in the highest components of the superfields $\bar{R}R$, $(W_{\alpha\beta\gamma})^2$ and $(G_{\alpha\dot{\alpha}})^2$ respectively. One can also define the superGauss-Bonnet combination

$$SGB = 8(W_{\alpha\beta\gamma})^2 + (\bar{\mathcal{D}}^2 - 8R)(G^2_{\alpha\dot{\alpha}} - 4\bar{R}R) \tag{6}$$

The bosonic Gauss-Bonnet term is contained in the highest chiral component of SGB. It follows that the most general quadratic supergravity Lagrangian is given by

$$\begin{aligned} \mathcal{L}_Q &= \int d^4\theta E \Sigma(\bar{\Phi}_i, \Phi_i)\bar{R}R + \int d^4\theta \frac{E}{R} g(\Phi_i)(W_{\alpha\beta\gamma})^2 \\ &+ \int d^4\theta E \Delta(\bar{\Phi}_i, \Phi_i)(G_{\alpha\dot{\alpha}})^2 + hc \end{aligned} \tag{7}$$

# 3 (2,2) Symmetric $Z_N$ Orbifolds

Although our discussion is perfectly general, we will limit ourselves to orbifolds, such as $Z_4$, which have (1,1) moduli only. The relevant superfields are the dilaton, S, the diagonal moduli $T^{II}$, which we'll denote as $T^I$, and all other moduli and matter superfields, which we denote collectively as $\phi^i$. The associated Kahler potential is

$$\begin{aligned} K &= K_0 + Z_{ij}\bar{\phi}^i\phi^j + \mathcal{O}((\bar{\phi}\phi)^2) \\ \kappa^2 K_0 &= -ln(S+\bar{S}) - \sum (T^I + \bar{T}^I) \\ Z_{ij} &= \delta_{ij} \prod (T^I + \bar{T}^I)^{q^i_I} \end{aligned} \tag{8}$$

The tree level coupling functions $f_{ab}$ and g can be computed uniquely from amplitude computations and are given by

$$\begin{aligned} f_{ab} &= \delta_{ab} k_a S \\ g &= S \end{aligned} \tag{9}$$

There is some ambiguity in the values of $\Delta$ and $\Sigma$ due to the ambiguity in the definition of the linear supermultiplet. We will take the conventional choice

$$\Delta = -S \qquad (10)$$
$$\Sigma = 4S$$

It follows that, at tree level, the complete $Z_N$ orbifold Lagrangian is given by $\mathcal{L} = \mathcal{L}_E + \mathcal{L}_Q$ where

$$\mathcal{L}_Q = \frac{1}{4}\int d^4\theta \frac{E}{R} SSGB + hc \qquad (11)$$

Using this Lagrangian, we now compute the one-loop moduli-gravity-gravity anomalous threshold correction [5]. This must actually be carried out in the conventional (string frame) superspace formalism and then transformed to Kahler superspace [6]. We also compute the relevant superGreen-Schwarz graphs. Here we will simply present the result. We find that

$$\mathcal{L}^{1-loop}_{massless} = \frac{1}{24(4\pi)^2}\Sigma\Big[h^I \int d^4\theta(\bar{\mathcal{D}}^2 - 8R)\bar{R}R\frac{1}{\partial^2}D^2 ln(T^I + \bar{T}^I)$$
$$+ (b^I - 8p^I)\int d^4\theta (W_{\alpha\beta\gamma})^2 \frac{1}{\partial^2}D^2 ln(T^I + \bar{T}^I) \qquad (12)$$
$$+p^I \int d^4\theta(8(W_{\alpha\beta\gamma})^2 + (\bar{\mathcal{D}}^2 - 8R)((G_{\alpha\dot{\alpha}})^2 - 4\bar{R}R))\frac{1}{\partial^2}D^2 ln(T^I + \bar{T}^I) + hc\Big]$$

where

$$h^I = \frac{1}{12}(3\gamma_T + 3\vartheta_T q^I + \varphi)$$
$$b^I = 21 + 1 + n^I_M - dimG + \Sigma(1 + 2q^i_I) - 24\delta^I_{GS} \qquad (13)$$
$$p^I = -\frac{3}{8}dimG - \frac{1}{8} - \frac{1}{24}\Sigma 1 + \xi - 3\delta^I_{GS}$$

The coefficients $\gamma_T$ and $\vartheta_T$, which arise from moduli loops, and $\varphi$ and $\xi$, which arise from gravity and dilaton loops, are unknown. However, as we shall see, it is not necessary to know their values to accomplish our goal. Now note that if $h^I \neq 0$ then there are non-vanishing $\mathcal{R}^2$ terms in the superstring Lagrangian. If $b^I - 8p^I \neq 0$ then the Lagrangian has $C^2$ terms. Coefficient $p^I \neq 0$ merely produces a Gauss-Bonnet term. With four unknown parameters what can we learn? The answer is,a great deal! Let us take the specific

example of the $Z_4$ orbifold. In this case, the Green-Schwarz coefficients are known citeG

$$\delta_{GS}^{1,2} = -30, \delta_{GS}^{3} = 0 \tag{14}$$

which gives the result

$$b^{1,2} = 0, b^3 = 11 \times 24 \tag{15}$$

Now, let us try to set the coefficients of the $(C_{mnpq})^2$ terms to zero simultaneously. This implies that

$$b^I = 8p^I \tag{16}$$

for I=1,2,3 and therefore that

$$p^{1,2} = 0, p^3 = 33 \tag{17}$$

From this one obtains two separate equations for the parameter $\xi$ given by

$$\xi = \frac{3}{8}dimG + \frac{1}{8} + \frac{1}{24}\Sigma 1 - 90 \tag{18}$$

for I=1,2 and

$$\xi = \frac{3}{8}dimG + \frac{1}{8} + \frac{1}{24}\Sigma 1 \tag{19}$$

for I=3. Clearly these two equations are incompatible and, hence, it is impossible to have all vanishing $(C_{mnpq})^2$ terms in the 1-loop corrected Lagrangian of $Z_4$ orbifolds. We find that the same results hold in other orbifolds as well.

# 4 Conclusion

We conclude that non-vanishing $(C_{mnpq})^2$ terms are generated by light field loops in the N=1, D=4 Lagrangian of $Z_N$ orbifold superstrings. It is conceivable that loops containing the heavy tower of states might cancel these terms, but we find no reason, be it duality or any other symmetry, for this to be the case [8]. This is presently being checked by a complete genus-one string calculation [9]. We conjecture that cancellation will not occur. Unfortunately, since $\gamma_T$, $\vartheta_T$ and $\varphi$ are unknown, we can say nothing concrete about the existence of $\mathcal{R}^2$ terms in the Lagrangian. This issue will also be finally resolved in the complete superstring calculation.

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